ANGLE (EXPONENTIAL) MODULATION

Basic Concepts

__Angle Modulation__ either the phase or the frequency of the carrier wave is varied by $m(t)$, while the amplitude of the carrier wave is maintained as constant. In general, we define an angle modulated waveform by the expression

$$\varphi(t) = A_c \cos \theta(t)$$

where A_c = constant, and $\theta(t) = f[m(t)]$.
Observe that $\varphi(t)$ represents a rotating phasor of amplitude (= length) A_c and generalized angle $\theta(t)$ with instantaneous frequency given by:

$$\omega_i(t) = \frac{d}{dt}\theta(t)$$

or $$f_i(t) = \frac{1}{2\pi}\omega_i(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

Let us consider some specific cases:

__Unmodulated carrier__

$$\theta(t) = \omega_c t + \theta_0 \Rightarrow \omega_i(t) = \omega_c \quad \text{or} \quad f_i(t) = f_c$$

where θ_0 is an arbitrary initial phase term. Hence $\varphi(t)$ represents a phasor rotating at __constant__ angular velocity of ω_c.

__Phase modulation__ (PM)

$$\theta(t) \sim m(t) \Rightarrow \theta(t) = \omega_c t + \cancel{\theta_0}^{\,0} + K_p m(t)$$

such that

$$\omega_i(t) = \omega_c + K_p \frac{dm(t)}{dt}.$$

where we assumed with loss of generality that $\theta_0 = 0$.

Frequency Modulation (FM)

$\omega_i(t)$ is varied linearly with $m(t)$

$$\omega_i(t) \sim m(t)$$

$$\omega_i(t) = \omega_c + k_f m(t)$$

$$\theta(t) = \int_0^t \omega_i(\lambda)\, d\lambda$$

$$= \int_0^t [\omega_c + k_f m(\lambda)]\, d\lambda$$

$$= \omega_c t + k_f \int_0^t m(\lambda)\, d\lambda .$$

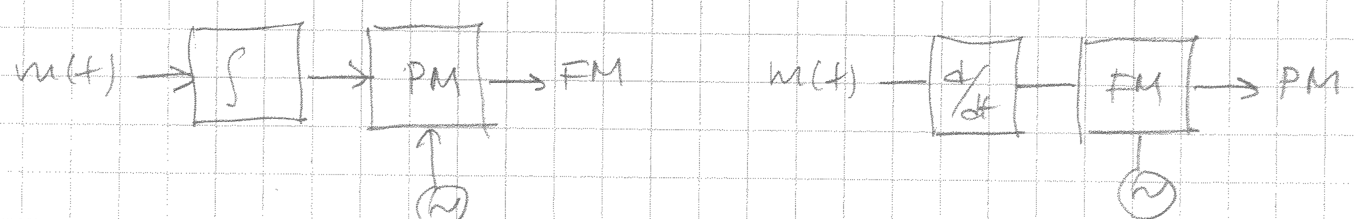
$$\varphi_{FM}(t) = A_c \cos\left(\omega_c t + k_f \int_0^t m(\lambda)\, d\lambda\right).$$

We can consider FM as a special case of PM where

$$\theta(t) \sim k_f \int_0^t m(\lambda)\, d\lambda$$

or PM as a special case of FM where

$$\omega_i(t) \sim \frac{d}{dt} m(t)$$



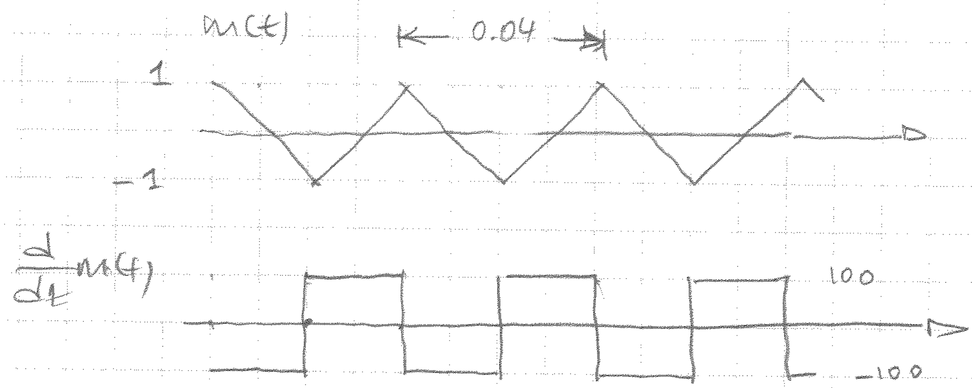
Hence, we need to study only one. Properties of the other can be easily obtained from the other. FM is by far the most common one. Therefore, we will study FM.

EX Let $m(t)$ be the time waveform as shown in the figure below. Determine the corresponding PM and FM waveforms

Also, let $K_f = 2\pi = K_p$ and $f_c = 1000$ Hz

$$\varphi_{PM}(t) = A_c \cos(\omega_c t + K_p m(t) + \cancelto{0}{\theta_0})$$

$$\varphi_{FM}(t) = A_c \cos(\omega_c t + k_f \int_0^t m(\lambda)\, d\lambda + \cancelto{0}{\theta_0})$$



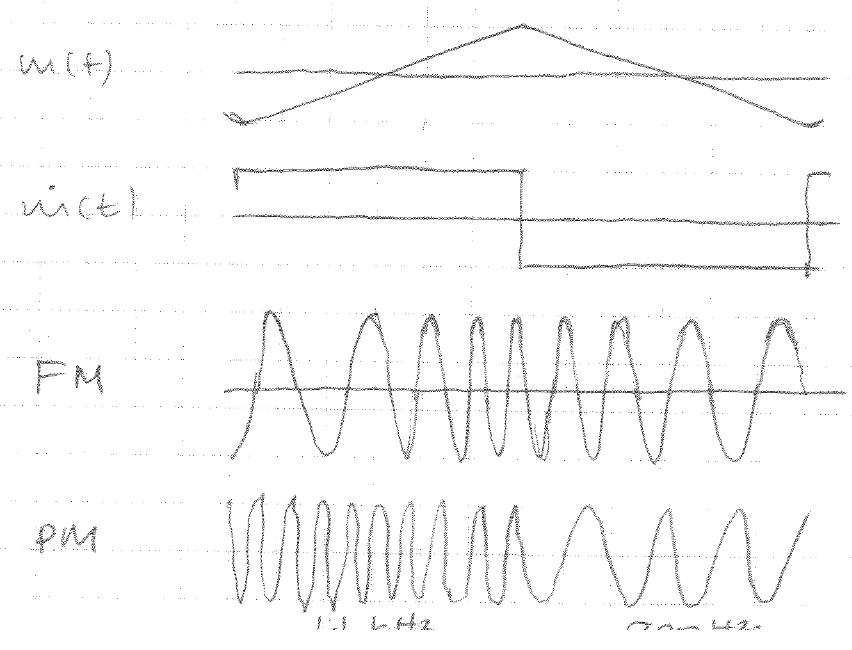
slope $= \pm \frac{2}{0.02} = 10$

Observe that for PM $\omega_i = \frac{d\theta}{dt} = \omega_c + k_p \frac{d}{dt} m(t)$

FM $\omega_i = \omega_c + k_f m(t)$

or equivalently PM $f_i = \frac{1}{2\pi}\omega_i = f_c + \frac{k_p}{2\pi}\left[\frac{dm(t)}{dt}\right]$

FM $f_i = f_c + \frac{k_f}{2\pi} m(t)$



$\boxed{\text{FM}}$ In particular, we want to study the spectrum of an FM signal. Let us start with a simple case:

$$m(t) = A_m \cos \omega_m t$$

$$\omega_i(t) = \omega_c + K_f\, m(t) = \omega_c + K_f A_m \cos \omega_m t$$

$$= \omega_c + \Delta\omega \cos \omega_m t \quad \text{with } \Delta\omega = K_f A_m$$

Here, $\Delta\omega$ (or the related $\Delta f = \Delta\omega/2\pi$) represents the maximum frequency deviation of f_i from f_c. Observe that

$$\Delta f \sim A_m = \max_t [m(t)]$$

and is independent of f_m.

$$\omega_i(t) = \omega_c + \Delta\omega \cos \omega_m t$$

$$\theta(t) = \int_0^t \omega_i(\lambda)\, d\lambda$$

$$= \omega_c t + \Delta\omega \int_0^t \cos \omega_m t$$

$$= \omega_c t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t$$

$$= \omega_c t + \beta \sin \omega_m t$$

with $\beta = \Delta\omega/\omega_m = \Delta f / f_m$. Therefore, we have

$$\varphi_{FM}(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t)$$

$$= A_c \cos \omega_c t \cos(\beta \sin \omega_m t) - A_c \sin \omega_c t \sin(\beta \sin \omega_m t)$$

<u>CASE 1</u>: Narrowband FM $\Rightarrow$ β is small (typically $\beta < 0.3$)

$$\cos(\beta \sin \omega_m t) \approx 1$$

and

$$\sin(\beta \sin \omega_m t) \approx \beta \sin \omega_m t.$$

Hence, for NBFM (and for $m(t) = A_m \cos w_m t$)

$$\varphi_{FM}(t) \approx A_c \cos w_c t - A_c \beta \sin w_c t \sin w_m t.$$

If we compare the above expression with AM

$$\varphi_{AM}(t) = A_c [1 + \mu \cos w_m t] \cos w_c t$$

$$= A_c \cos w_c t + A_c \mu \cos w_c t \cos w_m t.$$

$\Rightarrow$ $\boxed{\beta = \text{Modulation index}}$

Usually, if $\beta < 0.3 \Rightarrow$ NBFM.

Although AM and NBFM have similarities, they are fundamentally and distinctively different methods of modulation.

$\boxed{\text{AM}}$ $\varphi_{AM} = A_c [1 + \mu \cos w_m t] \cos w_c t$

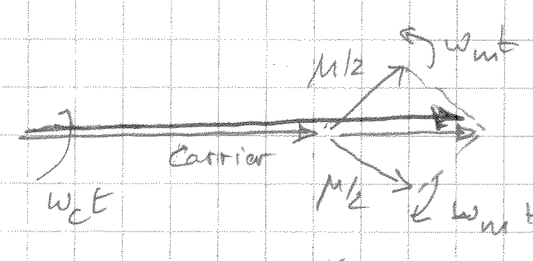
$$= \mathrm{Re}\left\{ A_c \left[1 + \frac{\mu}{2} e^{j\omega_m t} + \frac{\mu}{2} e^{-j\omega_m t}\right] e^{j\omega_c t} \right\}$$

$$\varphi_{FM}(t) = A_c [\cos w_c t - \beta \sin w_m t \sin w_c t]$$

$$= \mathrm{Re}\left\{ A_c [1 + j\beta \sin w_m t] e^{j w_c t} \right\}$$

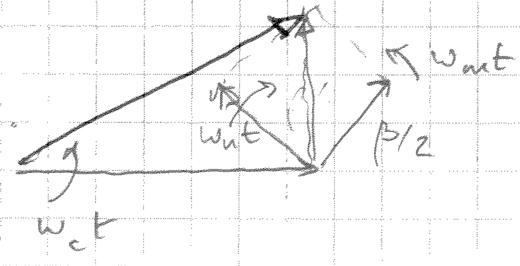
$$= \mathrm{Re}\left\{ A_c \left[1 + \frac{\beta}{2} e^{j\omega_m t} - \frac{\beta}{2} e^{-j\omega_m t}\right] e^{j w_c t} \right\}$$

AM.

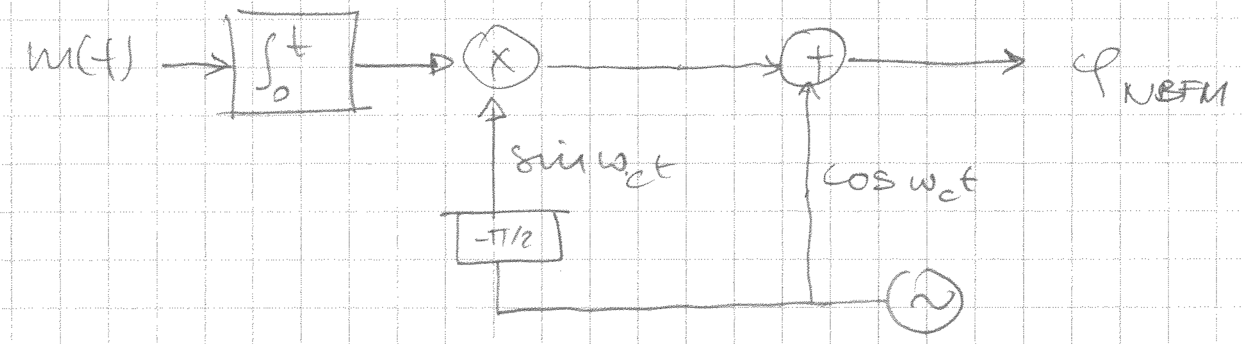


adds in-phase

FM



adds in-quadrature with the carrier. NBFM gives rise to phase variations with very little amplitude change.



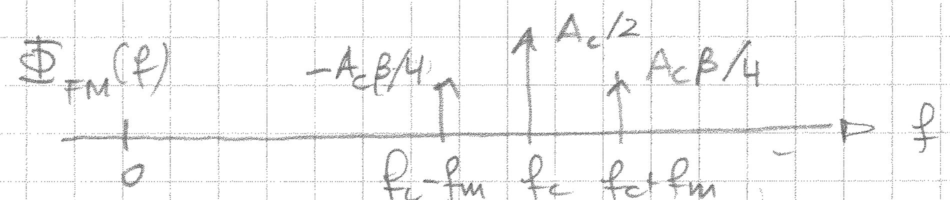
In FM $\left[f_i \text{ of } \varphi_{FM} \right] \sim m(t)$

But $f_i \sim A_m \not\Rightarrow \Phi_{FM} \sim A_m$

f_i and "f" in FT are different !!

Spectrum: $\varphi_{FM} = A_c \cos \omega_c t - A_c \beta \sin \omega_m t \sin \omega_c t$

$$\varphi_{FM}(t) = A_c \cos \omega_c t - \frac{A_c \beta}{2} \left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t \right].$$



$|\leftarrow B_T \rightarrow| = 2 f_m$ Same as in AM.

Now, let us consider a more general case with

$$a(t) = \int_0^t m(\lambda)\, d\lambda .$$

we then have:

$$\begin{aligned}
\varphi_{FM}(t) &= A_c \cos\left(\omega_c t + k_f a(t) \right) \\
&= Re \left\{ A_c \exp\left[j\omega_c t + j k_f a(t) \right] \right\} \\
&= Re \left\{ A_c \left[1 + j k_f a(t) - k_f^2 \frac{a^2(t)}{2!} + \cdots \right] e^{j\omega_c t} \right\} \\
&= A_c \cos \omega_c t - k_f a(t) \sin \omega_c t - \frac{k_f^2 a^2(t)}{2!} \cos \omega_c t \cdots
\end{aligned}$$

Remember that

if $m(t)$ is bandlimited to B-Hz

$a(t)$ " " " " $\left(\int_0^t \text{ is linear} \right)$

$a^2(t)$ " " " $2B$-Hz

$\vdots$

$a^n(t)$ " " " nB-Hz.

Theoretically $\varphi_{FM}(t)$ is <u>not bandlimited.</u>
However, if NBFM $\Rightarrow k_f$ is small.

$$\Rightarrow \varphi_{NBFM}(t) \approx A_c \cos \omega_c t - k_f A_c a(t) \sin \omega_c t$$

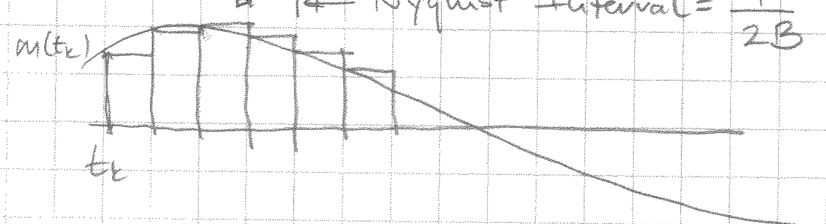
Bandwidth:

$$B_{\varphi_{NBFM}} = 2 \text{ Bandwidth } [a(t)]$$
$$= 2 \text{ Bandwidth } [m(t)].$$

If, on the other hand, k_f is not small, i.e. we are not in the NBFM territory and we can no longer ignore the higher order terms in the expansion of $\varphi_{FM}(t)$.

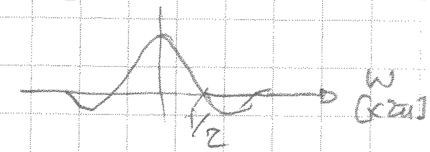
Let us consider a more general case.

$\rightarrow| \;|\leftarrow$ Nyquist Interval $= \frac{1}{2B}$

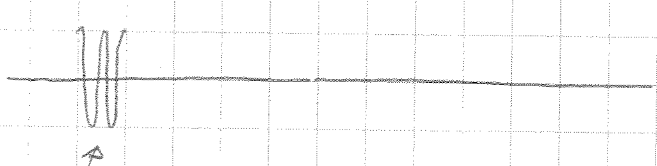


Recall

$$\text{rect}\left(\frac{t}{\tau}\right) \longleftrightarrow \tau \,\text{sinc}\left(\frac{\omega\tau}{2}\right)$$

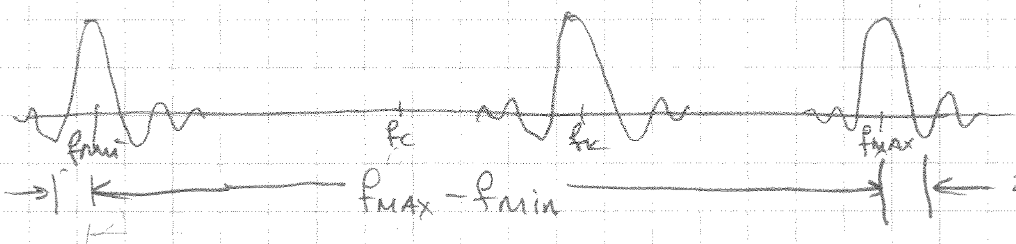


with $\tau = 1/2B$



Frequency $= f_c + k_f/2\pi \; m(t_k) = f_k$.

How does the corresponding spectrum look like.



$$f_{MAX} = f_c + \frac{k_f}{2\pi} \max_t [m(t)] = f_c + \frac{k_f}{2\pi} m_p$$

$$f_{min} = f_c + \frac{k_f}{2\pi} \min_t [m(t)] = f_c - \frac{k_f}{2\pi} m_p$$ (if $m(t)$ is symmetric

$$B_T \approx [f_{MAX} - f_{MIN}] + 2(2B)$$
$$= 2\frac{k_f}{2\pi} m_p + 4B$$
$$= 2\left(2B + \frac{k_f m_p}{2\pi}\right) = 2(2B + \Delta f)$$

where Δf = frequency deviation $= \frac{k_f m_p}{2\pi}$ Hz

This is an approximation to B_{T-FM} and not a very good one. In the case of NBFM

$$B_{NBFM} \approx 2B$$

$\Rightarrow$

$$B_{FM} = 2(B + \Delta f) \quad \text{Carson's Rule}$$

would be a better approximation to the bandwidth estimate of FM signals.

If we let $\beta = \Delta f / B = \Delta f / f_m = \Delta \omega / \omega_m$ modulation index for single tone mod

or $D = \Delta f / B$ = deviation index

Then
$$\boxed{\begin{aligned} B_T &\approx 2B(1+\beta) \\ &\approx 2B(1+D) \end{aligned}}$$

Determining a closed-form expression for the spectrum of φ_{FM} for arbitrary β is not possible. We will look at the single-tone modulation case.

$$m(t) = A_m \cos \omega_m t$$

$$a(t) = \int_0^t m(\lambda)\,d\lambda = \frac{A_m}{\omega_m} \sin \omega_m t.$$

such that

$$\varphi_{FM}(t) = A_c \cos\left(\omega_c t + k_f \int_0^t m(\lambda)\,d\lambda\right)$$

$$= A_c \cos\left(\omega_c t + \frac{k_f A_m}{\omega_m} \sin \omega_m t\right)$$

$$= A_c \cos(\omega_c t + \beta \sin \omega_m t) \quad \text{with } \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{2\pi f_m}$$

$$= Re\left\{ A_c e^{j\beta \sin \omega_m t} \cdot e^{j\omega_c t} \right\}.$$

observe that $e^{j\beta \sin \omega_m t}$ is periodic with period $2\pi/\omega_m$ therefore we can expand it in a Fourier series.

$$e^{j\beta \sin \omega_m t} = \sum_n C_n e^{jn\omega_m t}$$

with

$$C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t}\,dt.$$

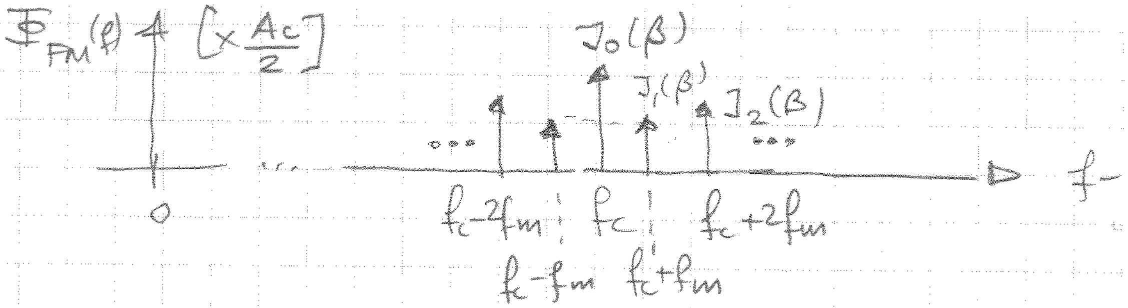
$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\,dx \quad \text{with } x = \omega_m t$$

$$= J_n(\beta)$$ = Bessel function of 1st kind (denoted by "J", order "n" and argument "β").

$$\varphi_{FM}(t) = Re\left\{ A_c \sum_n J_n(\beta) e^{j(\omega_c + n\omega_m)t} \right\}$$

$$= A_c \sum_n J_n(\beta) \cos(\omega_c + n\omega_m)t$$

$$\Phi_{FM}(f) = \frac{A_c}{2} \sum_n J_n(\beta)\left[\delta(f - (f_c + nf_m)) + \delta(f + (f_c + nf_m))\right]$$



Hence, the J_n's determine the overall shape of Φ_{FM}.

Observations

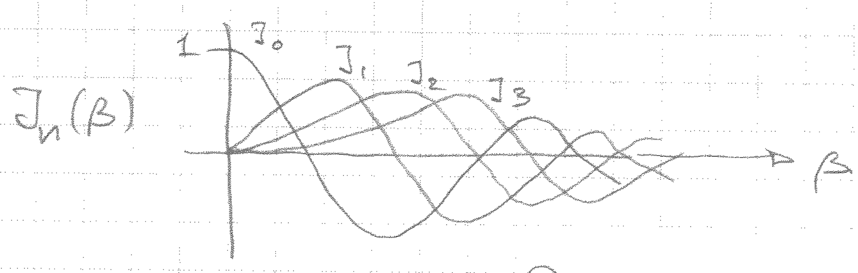
1. $J_n(\beta)$ are real valued.
2. $J_n(\beta) = J_{-n}(\beta)$ for n even (e.g. $J_4(\beta) = J_{-4}(\beta)$)

   $J_n(\beta) = -J_{-n}(\beta)$ for n odd (e.g. $J_5(\beta) = -J_{-5}(\beta)$)

   i.e., $J_n(\beta) = (-1)^n J_{-n}(\beta)$.
3. For β small ($\beta < 0.3$)

$$\left.\begin{aligned} &J_0(\beta) \approx 1 \quad J_1(\beta) = -J_{-1}(\beta) = \beta/2 \\ &J_n(\beta) \approx 0 \quad |n| \geq 2 \end{aligned}\right\} \text{NBFM}$$

4. $\sum_n J_n^2(\beta) = 1$.
5. J_n's are well tabulated.



Bessel functions are frequently encountered in a number of engineering problems. They are solutions to the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (\lambda^2 x^2 - n^2) = 0 \qquad \begin{array}{l}\text{Wave Equation} \\ \text{Heat Transfer.}\end{array}$$

Recall that "sin" and "cos" are also solutions to D.E.

$$\frac{d^2y}{dx^2} + \mu^2 y = 0.$$

EXAMPLES

① <u>NBFM</u> $(\beta < 0.3)$

$$J_0(\beta) \approx 1 \qquad J_1(\beta) = -J_{-1}(\beta) = \beta/2$$

$$J_n(\beta) \approx 0 \quad |n| \geq 2$$

$$\varphi_{FM}(t) = A_c \cos \omega_c t + \frac{A_c\beta}{2} \cos(\omega_c + \omega_m)t - \frac{A_c\beta}{2} \cos(\omega_c - \omega_m)t$$

As before NBFM = carrier + 2 sidebands.

② <u>WBFM</u>

Observations:

- FM with sinusoidal modulation results in an infinite number of sidebands at $f_c \pm n f_m$
- Magnitudes of sidebands decrease as $|f| >> f_c$ $(n \uparrow)$
- For practical purposes the power in φ_{FM} is contained within a finite bandwidth.
- Carrier amplitude $A_c J_0(\beta)$ depends on β.
- Total power in φ_{FM}

$$\overline{\varphi_{FM}^2} = \frac{A_c^2}{2} \sum_n J_n^2(\beta) = \frac{A_c^2}{2} \quad (\text{also from } \varphi_{FM} = A_c \cos(\ldots))$$

$$= \text{constant}.$$

But, P_{SB} and P_c change as a function of β.

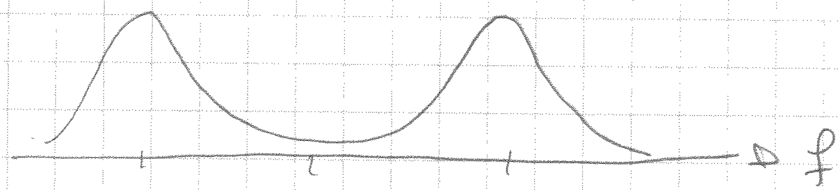
- $\beta = \Delta f / f_m$, we can change β by.

(i) Δf = constant. change f_m.
(ii) Δf changes and f_m = constant.

(i) Δf = constant $\beta \uparrow \Rightarrow f_m \downarrow 0$
$\Rightarrow$ more spectral lines are added within $2\Delta f$.

(ii) f_m = constant $\beta \uparrow \implies \Delta f \uparrow$
$\implies$ more spectral lines at constant spacing are added.

In the limiting case



$\longleftarrow 2\Delta f \longrightarrow$

$\longleftarrow B_T \approx 2B(1+\beta) \longrightarrow$ Carson's Rule

TRANSMISSION OF FM SIGNALS

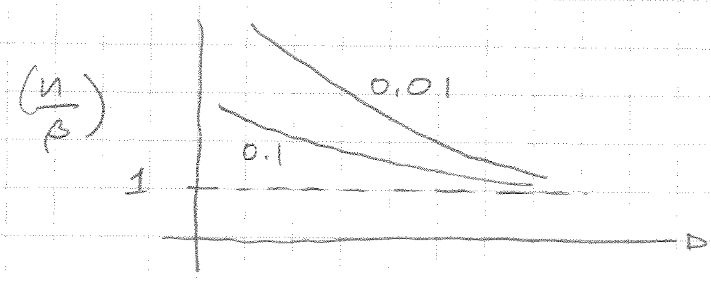
Questions: How many sidebands are significant to determine $B_{T\text{-}FM}$?

A rule commonly adapted is that a sideband is significant and its magnitude equals or exceeds 1% of the unmodulated carrier.

unmodulated carrier: $A_c J_0(\beta)$
other terms: $A_c J_n(\beta)$

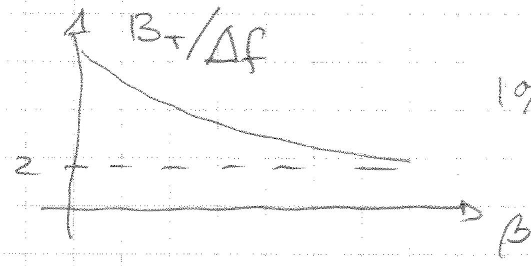
$$|J_n(\beta) A_c| \geq |A_c J_0(\beta)| \, 10^{-2}$$

$$\text{i.e.} \quad |J_n(\beta)| \geq 10^{-2} J_0(\beta)$$



0.01 : 1% criteria
0.1 : 10% criteria.

or the same information is translated onto a "universal curve"



1% rule.

CASE 1 β is very large

$$(n/\beta) \longrightarrow 1 \quad \text{i.e.,} \quad \boxed{n=\beta}$$

$$B_T = 2nf_m = 2\beta f_m = \boxed{2\Delta f}$$

or the same information can be read from the universal curve.

CASE 2 β is very small $\Rightarrow \boxed{n=1}$

$$\Rightarrow \boxed{B_T = 2f_m}$$

CASE 3 Arbitrary/intermediate β.

$$B_T = 2f_m + 2\Delta f = 2f_m(1 + \Delta f/f_m)$$
$$= 2f_m(1+\beta)$$
$$= 2B(1+\beta) \quad \text{same as Carson's Rule.}$$

Further observations about Carson's Rule

- approaches correct limits as $\beta \downarrow 0$ and $\beta \uparrow$
- B_T(Carson) < B_T(1%-rule) with max error for $\beta \approx 1$.
- Carson's rule displays the effects of the 2 mechanism in the generation of FM signals.
- Carson's rule is also valid for other bandlimited and finite power signals.

Let $m(t)$ bandlimited to B_m-Hz, then we have

$$\left.\begin{array}{l} f_m \longrightarrow B_m \\ \Delta\omega \longrightarrow k_f \max[m(t)] = k_f m_p \end{array}\right\} \beta = \frac{\Delta\omega}{\omega_m} = \frac{k_f m_p}{2\pi B_m}$$

$$B_T \approx 2B_m(1+\beta)$$

EXAMPLE Commercial FM Broadcasting.

$$\left.\begin{array}{l}\Delta f = 75 \text{ kHz} \\ B_M = 15 \text{ kHz}\end{array}\right\} \beta = \Delta f / B_m = 5$$

Carson's Rule : $B_T \approx 2(75+15) = \boxed{180 \text{ kHz}}$

1%-Rule : From the universal curve corresponding to $\beta = 5$

$$\frac{B_T}{\Delta f} \approx 3.2$$

$$B_T = (3.2)\Delta f = (3.2)(75) \text{ kHz}.$$
$$= \boxed{240 \text{ kHz}}$$

Immunity of Angle Modulated Signals to Nonlinearities

$\varphi_{FM}(t)$ (also $\varphi_{PM}(t)$, but we focus our efforts on FM) is a constant amplitude signal which makes it less susceptible to nonlinearities

For example

modulated signal $\longrightarrow$ [System Non-linearity Model] $\longrightarrow$ output

Let:

$$o/p = a[i/p] + b[i/p]^2 + c[i/p]^3 + \dots$$

$\boxed{AM}$ $[i/p] = m(t)\cos\omega_c t = m \cdot \cos(\,)$

$$[o/p] = a \cdot m \cdot \cos(\,) + b\,m^2 \cos^2(\,) + c\,m^3 \cos^3(\,)$$

$$= a \cdot m \cdot \cos(\,) + \frac{b}{2} m^2 + \frac{b}{2} m^2 \cos 2\omega_c t$$

$$+ \frac{3}{4} c\,m^3 \cos(\,) + \frac{c}{4} m^3 \cos 3\omega_c t$$

$$= \left[\frac{b}{2}\; m(t)\right] \qquad \text{baseband}$$

$$+ \left[a\,m(t) + \frac{3}{4} c\,m^3(t)\right] \cos\omega_c t \qquad \pm\omega_c$$

$$+ \left[\frac{b}{2} m^2(t)\right] \cos 2\omega_c t \qquad \pm 2\omega_c$$

$$+ \left[\frac{c}{4} m^3(t)\right] \cos 3\omega_c t \qquad \pm 3\omega_c$$

Even after filtering the envelope of the signal $\pm\omega_c$ will remain as $[a\,m(t) + 3/4\, c\,m^3(t)] \not\propto m(t)$.

$\boxed{\text{FM}}$ $[i/p] = \cos(w_c t + \varphi(t))$ with $\varphi(t) = K_f \int_0^t m(\lambda)\,d\lambda$.

$$[o/p] = a\cos(\;) + b\cos^2(\;) + c\cos^3(\;)$$

$$= a\cos(\;) + \frac{b}{2} + \frac{b}{2}\cos(2w_c t + 2\varphi)$$

$$+ \frac{3}{4}c\cos(\;) + \frac{c}{4}\cos(3w_c t + 3\varphi)$$

$$= \left[\frac{b}{2}\right]$$

$$+ \left[a + \frac{3}{4}c\right]\cos(w_c t + \varphi(t))$$

$$+ \left[\frac{b}{2}\right]\cos(2w_c t + 2\varphi(t))$$

$$+ \left[\frac{c}{4}\right]\cos(3w_c t + 3\varphi(t))$$

After BPF centered at $\pm w_c$ we will have

$$\left[a + \frac{3}{4}c\right]\cos(\underbrace{w_c t + \varphi(t)}_{\text{information is intact.}})$$

We will further analyze the performance of angle modulated signals, particularly in comparison with amplitude modulated signal

GENERATION OF FM WAVES

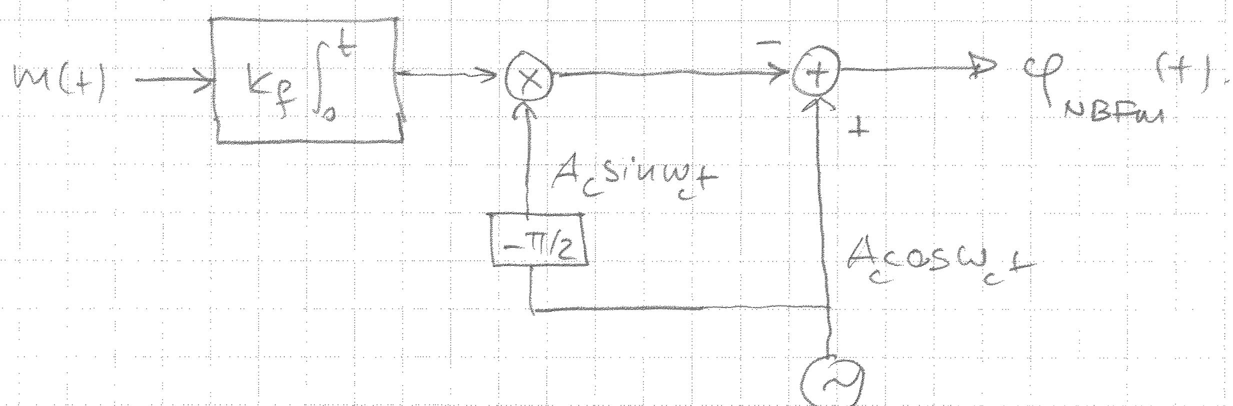
Two major techniques: (1) Indirect Method
(2) Direct Method (read from book)

(1) Indirect method

$$m(t) \longrightarrow NBFM \longrightarrow WBFM$$

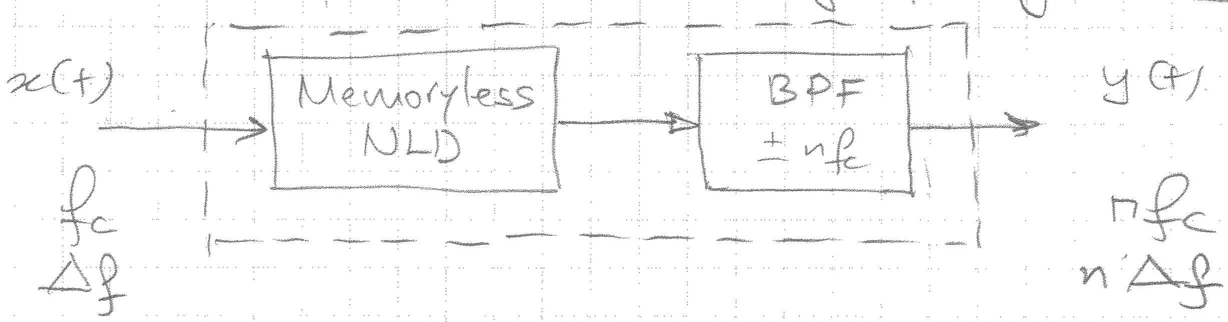
Step 1: $m(t) \longrightarrow NBFM$

$$\varphi_{NBFM}(t) = A_c \cos \omega_c t - A_c \left[k_f \int_0^t m(\lambda)\, d\lambda \right] \sin \omega_c t$$



Step 2 NBFM → WBFM

For this operation we use a frequency multiplier



$$y(t) = \left[a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \dots \right] * h_{BPF}(t)$$

Let us consider the case, where

NLD: $y(t) = a_2 x^2(t)$

$$y(t) = a_2 \cos^2(\omega_c t + \varphi(t)) * h_{BPF}(t)$$

$$= \left[\frac{a_2}{2} + \frac{a_2}{2}\cos(2\omega_c t + 2\varphi(t))\right] * h_{BPF}$$ tuned to $\pm 2f_c$

$$= \frac{a_2}{2}\cos(2\omega_c t + 2\varphi(t))$$

Thus, both the carrier frequency and the modulation/dispertion index have been doubled. If we generalize the results of this example to an n-th order nonlinearity.

$$[a_n x^n(t)] * h_{BPF}(t) \longrightarrow \begin{Bmatrix} n f_c \\ n\beta \text{ or } n\Delta f \end{Bmatrix}$$

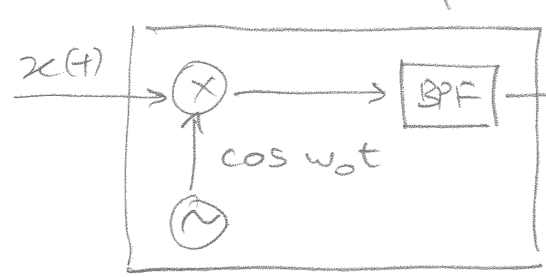
f_c and $\beta/\Delta f$ tuned to nf_c

<u>Remark:</u> if f_m / B_m is fixed and $\beta = \Delta f / B_m$, then $\beta \to n\beta$ or $\Delta f \to n\Delta f$.

In practise very abrupt non-linearities can be generator using diodes ... With proper design techiques $n \sim 1000$

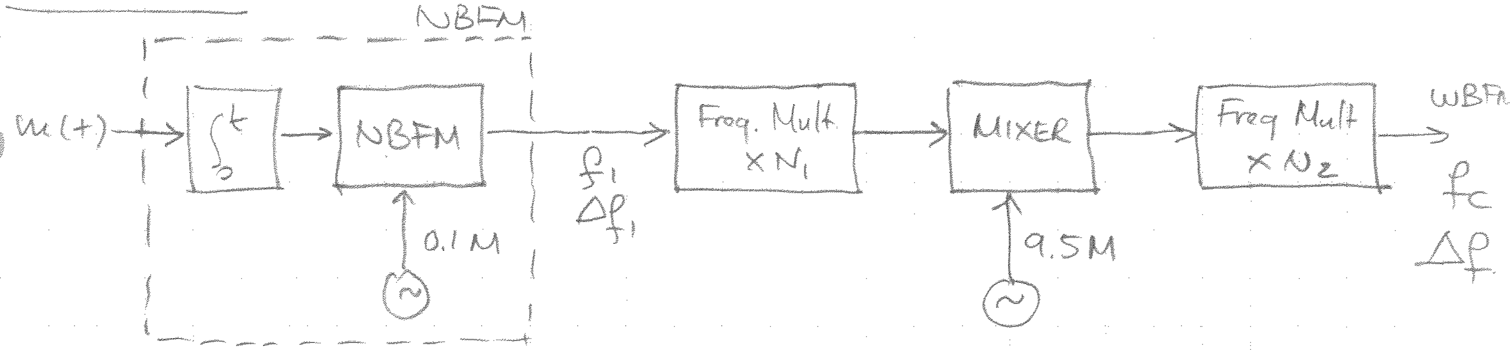
Use of a frequency multiplier increases the carrier frequency AND Δf. To avoid this frequency converters (i.e., mixers) are often used to control f_c.

MIXER
Freq. Conv.



$y(t) =$
$Y(\omega) = X(\omega \pm \omega_0)$
but otherwise no change in the shape of the spectrum.

EXAMPLE:



Given: * $m(t)$ bandlimited to [100 Hz, 15 kHz].

* NBFM: $f_1 = 0.1$ MHz WBFM: $f_c = 100$ MHz

$\Delta f_1 = 20$ Hz. $\Delta f = 75$ kHz

* MIXER : down mixer

For NBFM $\beta_1 = \dfrac{\Delta f_1}{15{,}000} = \dfrac{20}{15{,}000}$ = very small

or $\beta_1 = \dfrac{20}{100} = 0.5$ worst case scenario.

Thus starting with $(\Delta f_1)_{min} = 20$ Hz, we want

$$\left.\begin{array}{l}(\Delta f_1) = 20 \text{ Hz} \\ f_1 = 0.1 \text{ MHz}\end{array}\right\} \longrightarrow \left\{\begin{array}{l}\Delta f = 75 \text{ kHz} \\ f_c = 100 \text{ MHz}\end{array}\right.$$

To determine the overall FREQ. MULT. factor we consider the increase in Δf from 20 Hz to 75 kHz. This is simply due to the fact that FREQ. MIXING affects only f_c and not Δf.

$$N_1 N_2 = \frac{\Delta f}{[\Delta f_1]_{min}} = \frac{75{,}000}{20} = 3750.$$

After 1st FREQ. MULTIPLIER

$$\left.\begin{matrix}\Delta f_1\\ f_1\end{matrix}\right\} \xrightarrow[\times N_1]{} \left\{\begin{matrix}N_1\Delta f_1\\ N_1 f_1\end{matrix}\right.$$

After MIXER

$$\left.\begin{matrix}N_1\Delta f_1\\ N_1 f_1\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}N_1\Delta f_1\\ f_2 = 9.5\,\text{MHz} - N_1 f_1 \quad \text{(down-mixer)}\end{matrix}\right.$$

After 2nd FREQ. MULTIPLIER

$$\left.\begin{matrix}N_1\Delta f_1\\ f_2\end{matrix}\right\} \xrightarrow[\times N_2]{} \left\{\begin{matrix}N_1 N_2\Delta f_1\\ N_2 f_2\end{matrix}\right.$$

$$N_1 N_2 \Delta f_1 = 75{,}000 \implies N_1 N_2 = 3750$$

$$N_2 f_2 = N_2[9.5 - N_1 f_1] = 100$$

$$9.5 N_2 - N_1 N_2\, 0.1 = 100$$

$$9.5 N_2 - (3750)(0.1) = 100 \implies N_2 = \frac{100+375}{9.5}$$

$$\boxed{N_2 = 50}$$

$$N_1 = 3750/N_2 = 3750/50 = \boxed{75}$$

	O/P of NBFM	FREQ MULT 1	FREQ MIXER	FREQ. MULT 2
Δf	20 Hz	1500 Hz	1500 Hz	75 kHz
f_c	0.1 MHz	7.5 MHz	2.0 MHz	100 MHz

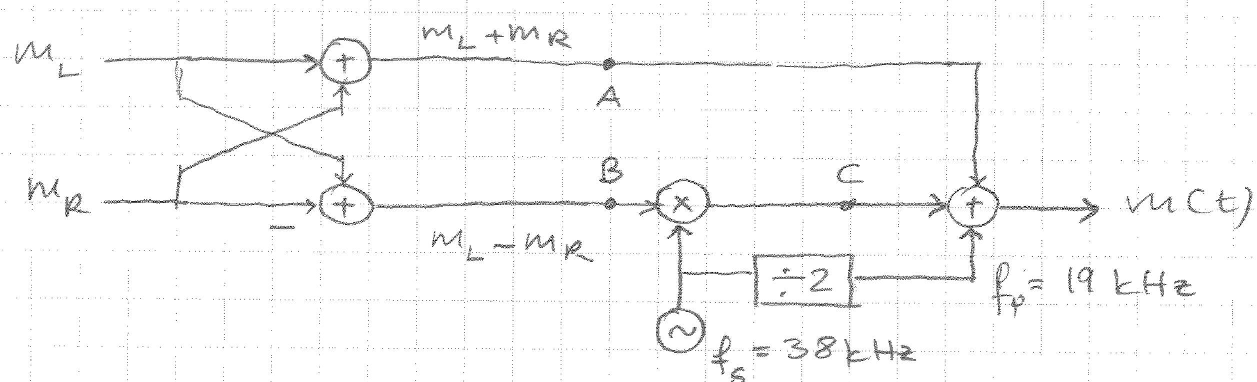
FM STEREO MULTIPLEXING

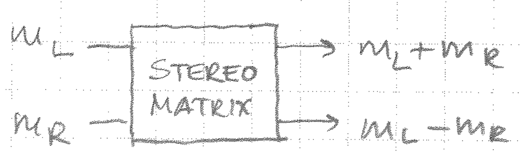
Let us consider FM stereo broadcasting in some detail.
Two important characteristics:
- stereo multiplexing is a form of FDM
- transmit two (and possibly three) signals via the same f_c.

In the design of the FM system, one has to take the following two factors into consideration

1. Transmission of stereo signal has to operate within the allocated FM broadcasting channel
2. Stereo receivers have to be backward compatible with monophonic receivers

Reminder: FM broadcasting $[\Delta f]_{MAX} = 75\text{ kHz}$ and
$B_m = 15\text{ kHz} \Rightarrow B_T \approx 2(\Delta f + B_m) = 180\text{ kHz}$
Regulations $\Rightarrow B_T = 200\text{ kHz}$



- 

$$\begin{bmatrix} m_L + m_R \\ m_L - m_R \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} m_L \\ m_R \end{bmatrix}$$

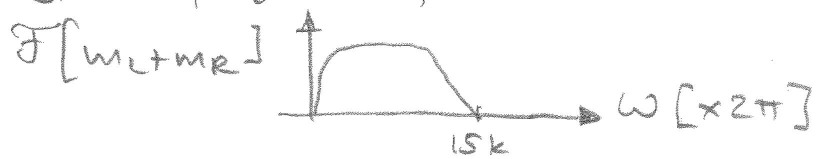
The stereo matrix has to be non-singular for proper de-multiplexing

- Baseband modulating signal

$$m(t) = [m_L + m_R] + [m_L - m_R]\cos \omega_{sc} t + K_p \cos \omega_p t$$

with f_{sc} = stereo subcarrier = 38 kHz, and
f_p = pilot (to indicate stereo transmission) = 19 kHz.

- A monophonic receiver will receive only $[m_L + m_R]$.
- At point A, the mono signal has a bandwidth of 15 kHz with a typical program spectrum

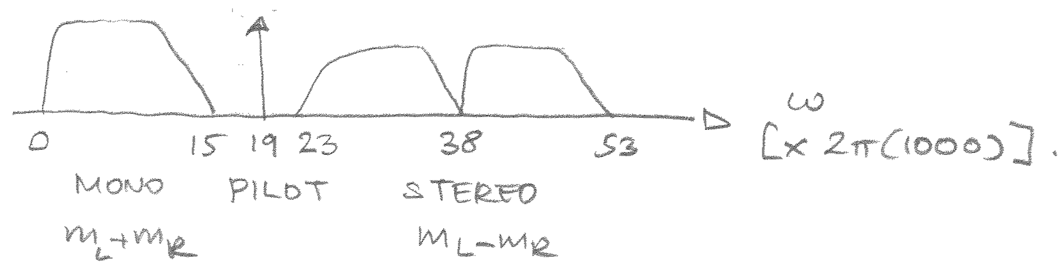


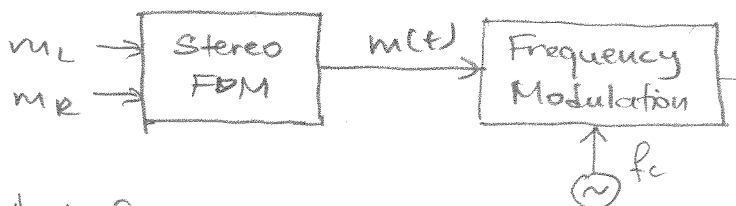
At point B we have the same type of spectrum [0, 15k]

At point C we have a DSB-SC signal with $f_c = f_{sc} = 38$ kHz

The pilot tone with $f_p = 19$ kHz is added for coherent demodulation of $[m_L - m_R]$. Hence the complete spectrum of the baseband signal $m(t)$ will be

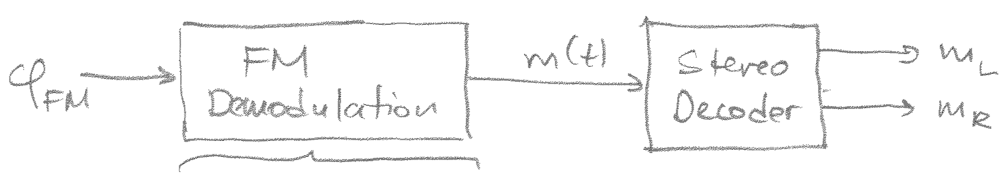
$M(\omega)$



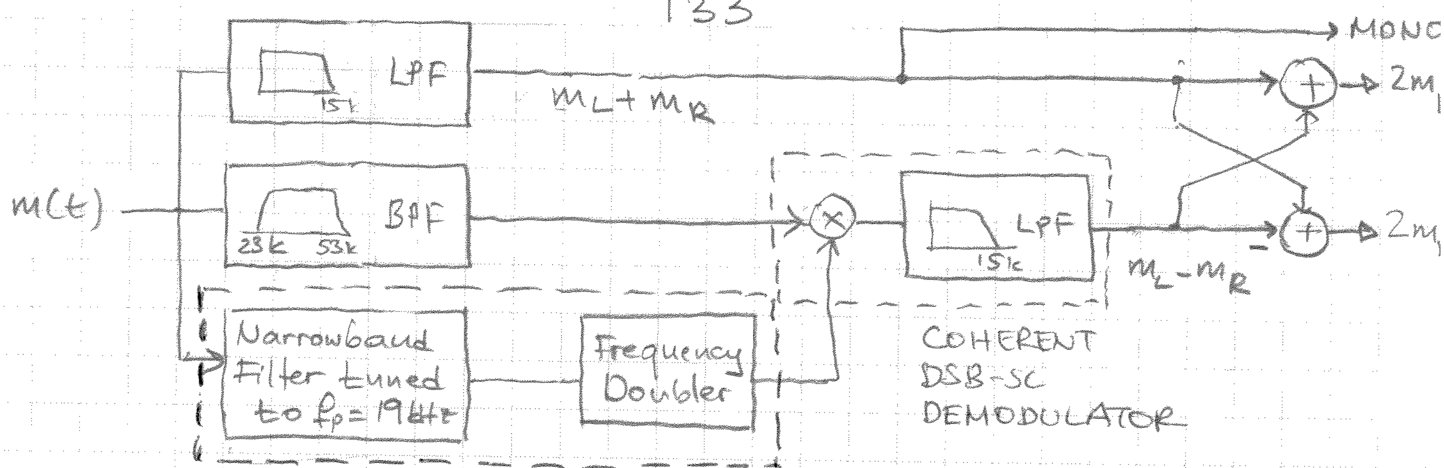


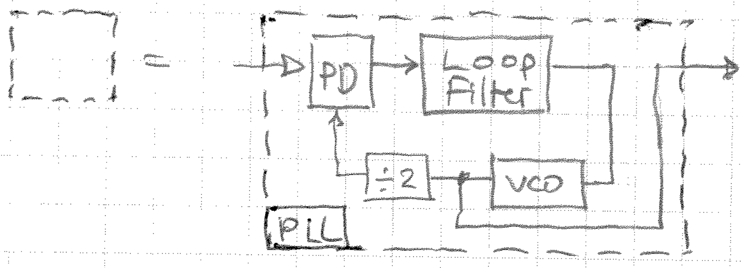
$$\varphi_{FM} = A_c \cos\left(\omega_c t + k_f \int_0^t m(\lambda)\, d\lambda\right)$$

And for the receiver



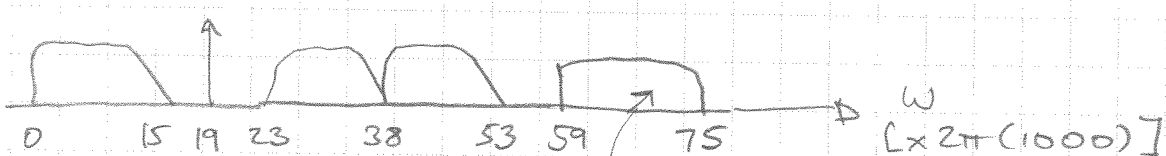
$\underbrace{\text{FM Demodulation}}$ Use a PLL tuned to f_c to extract $m(t)$ from φ_{FM}





VCO is tuned to 38 kHz
PLL will lock onto the 19 kHz pilot tone.

Actual baseband signal spectrum can have another component



SCA : Subsidiary communications authorization channel

For example SCA may contain background (MOZAK) music, older paging systems, alternate audio/language tracks. Hence, the baseband modulating signal will be of the form

$$m(t) = [m_L + m_R] + [m_L - m_R]\cos\omega_{sc} t + K_p \cos\omega_p t + K_S\, \varphi_{SCA}(t)$$

with φ_{SCA} being typically an FM signal with bw 5Hz.

$$\varphi_{FM}(t) = A_c \cos\left(\omega_c t + K_f \int_0^t m(\lambda)\, d\lambda\right)$$

$$= A_c \cos\left(\omega_c t + \boxed{\text{MONO}} + \boxed{\text{STEREO}} + \boxed{\text{PILOT}} + \boxed{\text{SCA}}\right)$$

As $m(t)$ has a baseband bandwidth of 75 kHz and for broadcasting $\Delta f_{peak} = 75$ kHz, this arrangement does not leave much room for modulation.

To ensure proper signal modulation, different parts of $m(t)$ are allocated certain portion of the total allowable Δf.

PILOT TONE $= 10\% \, [\Delta f]_{peak}$

Adjust its sensitivity parameter such that it will <u>not</u> result a frequency deviation of more than $[\Delta f]_{peak} \times 0.10$ i.e.

$$[\Delta f]_{PILOT} \leq 0.10 \, [\Delta f]_{peak} = 7.5 \text{ kHz}$$

$$\beta_{PILOT} = \frac{[\Delta f]_{PILOT}}{f_{PILOT}} = \frac{7.5}{19} = 0.395 \approx \text{NBFM}$$

If <u>MONO</u> station:

$\boxed{\text{MONO}}$	70 % of $[\Delta f]_{PEAK}$
$\boxed{\text{SCA}}$	30 % " "

If <u>STEREO</u> station:

$\boxed{\text{MONO}}$ + $\boxed{\text{STEREO}}$	80 %
$\boxed{\text{PILOT}}$	10 %
$\boxed{\text{SCA}}$	10 %

If STEREO & NO SCA

$\boxed{\text{MONO}}$ + $\boxed{\text{STEREO}}$	90 %
$\boxed{\text{PILOT}}$	10 %

Audio portion of the TV signal is also transmitted as an FM signal with $[\Delta f]_{PEAK} = 25$ kHz and a corresponding audio-FM bandwidth of $B_T \approx 2(\Delta f + B_M) \approx 80$ kHz

Some practical considerations in FM broadcasting

- FM envelope is constant
- Total signal power is constant and independent of β

$$P_{TOTAL} = A_c^2/2 = \frac{A_c^2}{2} \sum_{n=-\infty}^{+\infty} J_n^2(\beta) = \frac{A_c^2}{2} J_0^2(\beta) + P_{SB}.$$

- As P_{TOTAL} = constant, we have (from the properties of Bessel functions)

  $P_c \downarrow$ and $P_{SB} \uparrow$ with increasing β.

- Therefore to increase P_{SB} (useful signal power) we want to maximize β, but β values are limited by $[\Delta f]_{PEAK}$

- For mono signals this is not a problem as the signal has (almost) the full allocation of $[\Delta f]_{peak}$

- What further complicates the problem at hand is the dynamic nature of the music/audio signals
  For example @ $\beta = 100\%$ $[\Delta f]_{stereo} = (0.45)\,75{,}000 \approx 33$ kHz
  But we will achieve this only when the program signal hits a peak, which happens only few times during the program. The rest of the time we have to operate at much lower modulation levels (6-8%) which results in lower P_{SB}

- Typical average B_T of FM stations ~ 46 kHz whereas the maximum allowable B_T is 200 kHz.

- Solution (!?) compress program material, eliminate peaks, increase average signal amplitude, and therefore average Δf and average β.
  Stations with heavy compression $\Rightarrow B_T \approx 140-150$ kHz

  One station 70% average β but 2.5 dB dynamic range! In contrast a symphony orchestra has a dynamic range of ≈ 90 dB.

FM Broadcasting Standards.

Carrier Frequency Assignment	88.1 – 107.9 MHz in 0.2 MHz increments.
Channel BW	200 kHz.
Carrier Stability	$\pm$ 2 kHz of f_c
Peak Frequency Deviation	75 kHz
Audio Frequency Response	50 Hz to 15 kHz following a 75 μs pre-emphasis (NA) and 125 μs (Europe)
Modulation	$\beta \approx 5$ for $\Delta f = 75$ kHz and $f_m = B_m = 15$ kHz.
Maximum Licensed Power	100 kW.